

# Expected Behaviour Analysis of AVL Trees

Ricardo Baeza-Yates

Gaston H. Gonnet

Depto. de Cs. de la Computación  
Universidad de Chile  
Santiago, Casilla 2777  
Chile

Department of Computer Science  
University of Waterloo  
Waterloo, Ontario N2L3G1  
Canada

Nivio Ziviani

Depto. de Ciência da Computação  
Universidade Federal de Minas Gerais  
Belo Horizonte, Minas Gerais  
Brazil \*

## Abstract

In this paper we improve previous bounds on expected measures of AVL trees by using fringe analysis. A new way of handling larger tree collections that are not closed is presented. An inherent difficulty posed by the transformations necessary to keep the AVL tree balanced makes its analysis difficult when using fringe analysis methods. We derive a technique to cope with this difficulty obtaining the exact solution for fringe parameters even when unknown probabilities are involved. We show that the probability of a rotation in an insertion is between 0.37 and 0.73, that the fraction of balanced nodes is between 0.56 and 0.78, and that the expected number of comparisons in a search seems to be at most 12% more than in the complete balanced tree.

## 1 Introduction

Balanced tree structures are efficient ways of storing information. They provide an excellent solution for the dictionary data structure problem. For  $N$  elements the operations find, insert, and delete can be done in  $O(\log N)$  units of time. The most popular, for main memory, are AVL trees (also called Height Balanced trees).

AVL trees were introduced by Adel'son-Vel'skii and Landis in 1962 [AVL62]. A binary search tree is AVL if the height of the subtrees at each node differ by at most one. A balance field in each node can indicate this with two bits: +1, higher right subtree; 0, equal heights; -1, higher left subtree.

---

\*The work of the first author was also supported by the the Institute for Computer Research of the University of Waterloo, the second author by a Natural Sciences and Engineering Research Council of Canada Grant No. A-3353, and the third by a Brazilian Coordenação do Aperfeiçoamento de Pessoal de Nível Superior Contract No. 4799/77 and by the University of Waterloo.